ISI – Bangalore Center – B Math - Physics I –Back paper Exam
Date: 13 June 2019. Duration of Exam: 3 hours
Total marks: 90

Answer ALL questions

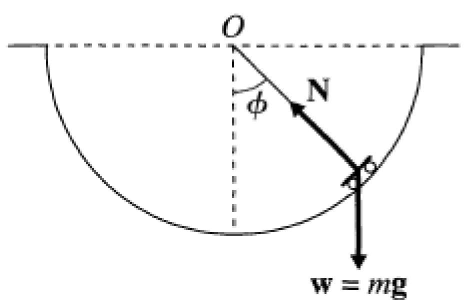
Q1: Total marks: 7+8=15

a.) For motion in two dimensions, show that the acceleration is given by

$$\mathbf{a} = \left(\ddot{r} - r\dot{\phi}^2\right)\hat{\mathbf{r}} + \left(r\ddot{\phi} + 2\dot{r}\dot{\phi}\right)\hat{\boldsymbol{\phi}}$$

where the symbols have their usual meanings.

b.) The figure below shows a small skateboard in a semicircular trough of radius ***R***. Find the equation of motion. Determine the period of oscillation if the angle is small.



Q2: Total marks: 10+5=15

a.)

The potential energy of a one-dimensional mass m at a distance r from the origin is

$$U(r) = U_o\left(\frac{r}{R} + \lambda^2\frac{R}{r}\right)$$

for $0 < r < \infty$, with U_o, R, and λ all positive constants. Find the equilibrium position r_o. Let x be the distance from equilibrium and show that, for small x, the PE has the form $U = \text{const} + \frac{1}{2}kx^2$. What is the angular frequency of small oscillations?

b.) Describe qualitatively (with the help of a sketch of potential energy vs r) the motion of the particle for the potential $U = U_0\left(\frac{R^2}{r^2} - \lambda^2\frac{R}{r}\right)$ if the particle starts at positive infinity and initially moving towards the origin.

Q3.Total Marks: 3+6+4+4+3=20

A particle of mass m is moving under the influence of a force $\vec{F} = -k\vec{r}$ where k is a positive constant and $\vec{r}$ is the position vector of the particle.
(a) Show that the motion of the particle lies in a plane.
(b) Assume without loss of generality that the motion is confined to the x-y plane. Find the position of the particle as a function of time, assuming that at $t = 0$, $x = a$, $y = 0$ and $v_x = 0$, $v_y = v_o$.
(c) Show that the orbit is an ellipse.
(d) Find the period of motion.
(e) Does the motion of the particle obey Kepler's Laws of planetary motion ?

Q4. Total Marks: 10+10=20

a.) Show that for a rigid body of mass M rotating with angular speed ω around a fixed axis going through its CM that is moving with velocity V , the total Kinetic energy is the sum of the kinetic energy of the CM ($MV^2/2$) plus the rotational energy ($I\omega^2$ /2)) where I is the Moment of Inertia around the axis of rotation.

Will this hold true if ω is time dependent?

b.)A uniform hollow cylinder of mass M and radius b is rolling without slipping down a rough plane inclined at an angle α to the horizon.

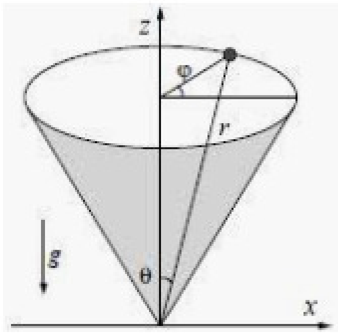
Show that the constraint forces do no work.

Apply the conservation of energy and show that the acceleration of the cylinder is (g sinα)/2, [The relevant moment of inertia is Mb^2]

If the hollow cylinder is not uniform, will it accelerate at a different rate? Please justify your answer. You need not do a detailed calculation to answer this.

Q6. Total Marks: 6+6+8=20

Consider a point particle of mass m sliding without friction inside a conical vase of opening angle θ whose axis is vertical. Use generalized coordinates (r,ϕ) to indicate the position of the particle where r is the perpendicular distance of the particle from the vertical axis and ϕ is the angle around the circle of radius r as shown in the figure below.



a.) Write the Lagrangian for the system and show that

$$mr^2 \sin^2\theta\, \dot{\phi} = \text{constant}$$

Let this constant be denoted by *J*. What does *J* physically mean?

b.)Show that the equation of motion for *r* can be written as

$$\ddot{r} = \frac{J^2}{m^2 r^3 \sin^2\theta} - g\cos\theta$$

c.) Find equilibrium value r_o of *r* and the frequency of small oscillations about r_o.